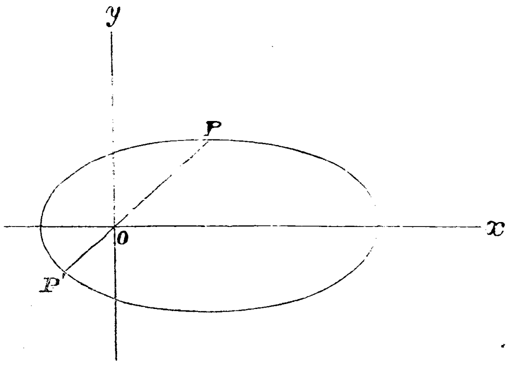
1875MNRAS..35..337C

surface-density, if accuracy is required, it will be necessary to include the square of a, and for small planets the term s (A) in (10); in the larger planets this will be unnecessary, but the cube of a must be retained.

On a Theorem in Elliptic Motion. By Prof. Cayley.

Let a body move through apocentre between two opposite points of its orbit, say from the point P, excentric anomaly u, to the point P′, excentric anomaly u', where u, u' are each positive, $u < \pi$, $u' > \pi$. Taking the origin at the focus, and the axis of x in the direction through apocentre, then—



Coordinates of P are $x = a\,(-\cos u + e)$, $y = a\sqrt{1-e^2}\,\sin u$.

,, P′ ,, $x = a\,(-\cos u' + e)$, $y = a\sqrt{1-e^2}\,\sin u'$;

whence, expressing that the points P, P′ are in a line with the focus,

$$\sin u'\,(-\cos u + e) - \sin u\,(-\cos u' + e) = 0,$$

that is

$$\sin(u'-u) = e\,(\sin u' - \sin u),$$

which is negative, viz. $u' - u$ is $> \pi$.

The time of passage from P to P′ is

$$\begin{aligned} n\,t &= (u' - e\sin u') - (u - e\sin u),\\ &= u' - u - e\,(\sin u' - \sin u),\\ &= u' - u - \sin(u' - u), \end{aligned}$$

which, $u' - u$ being greater than π, and $-\sin(u' - u)$ positive, is greater than π; viz. the time of passage is greater than one-half the periodic time. Of course if P, P′ are at pericentre and apocentre, the time of passage is equal one-half the periodic time.

The time of passage from P′ to P through pericentre is—

$$n\,\tau = 2\pi - (u' - u) + \sin(u' - u),$$

which is

$$= 2\pi - (u' - u) - \sin\{2\pi - (u' - u)\},$$

1875MNRAS..35..337C

where $2\pi - (u' - u)$, $= \alpha$ suppose, is an angle $< \pi$. Writing, then

$$nt = \alpha - \sin\alpha,$$

and comparing with the known expression for the time in the case of a body falling directly towards the centre of force, we see that the time of passage from P′ to P through pericentre, is equal to the time of falling directly towards the same centre of force, from rest at the distance $2a$ to the distance $a\,(1 + \cos\alpha)$, where, as above $\alpha = 2\pi - (u' - u)$, $u' - u$ being the difference of the excentric anomalies at the two opposite points P, P′. If $\alpha = \pi$, the times of passage are each $= \frac{\pi}{n}$, that is, one half the periodic time.

The foregoing equation $\sin(u' - u) = e\,(\sin u' - \sin u)$ gives obviously

$$\cos\frac{1}{2}(u'-u) = e\cos\frac{1}{2}(u'+u);$$

that is

$$1 + \tan\frac{1}{2}u\tan\frac{1}{2}u' = e\left(1 - \tan\frac{1}{2}u\tan\frac{1}{2}u'\right),$$

or

$$-\tan\frac{1}{2}u\tan\frac{1}{2}u' = \frac{1-e}{1+e};$$

(in the figure $\tan\frac{1}{2}u$ is positive, $\tan\frac{1}{2}u'$ negative); and we thence obtain further

$$\sin\frac{1}{2}(u'-u) = \cos\frac{1}{2}u'\cos\frac{1}{2}u\left(\tan\frac{1}{2}u' - \tan\frac{1}{2}u\right),$$

$$\sin\frac{1}{2}(u'+u) = \cos\frac{1}{2}u'\cos\frac{1}{2}u\left(\tan\frac{1}{2}u' + \tan\frac{1}{2}u\right),$$

$$\cos\frac{1}{2}(u'-u) = \cos\frac{1}{2}u'\cos\frac{1}{2}u\,.\,\frac{2e}{1+e},$$

$$\cos\frac{1}{2}(u'-u) = \cos\frac{1}{2}u'\cos\frac{1}{2}u\,.\,\frac{2}{1+e};$$

and thence also

$$\cos u + \cos u' = 2\cos\frac{1}{2}(u'+u)\cos\frac{1}{2}(u'-u),$$

$$= \cos^2\frac{1}{2}u'\cos^2\frac{1}{2}u\,.\,\frac{8e}{(1+e)^2}.$$

But we have

$$1 + \cos(u'-u) = 2\cos^2\frac{1}{2}(u'-u) = \cos^2\frac{1}{2}u'\cos^2\frac{1}{2}u\,.\,\frac{8e^2}{(1+e)^2},$$

1875MNRAS..35..339D

or comparing with the last equation,

$$1 + \cos(u' - u) = e(\cos u + \cos u'),$$

or, what is the same thing,

$$1 - \cos(u' - u) = (1 - e\cos u') + (1 - e\cos u);$$

and in like manner,

$$1 + \cos(u' + u) = 2\cos^2\frac{1}{2}(u' + u) = \cos^2\frac{1}{2}u'.\cos^2\frac{1}{2}u\frac{8}{(1+e)^2};$$

or comparing with the same equation,

$$1 + \cos(u' + u) = \frac{1}{e}(\cos u + \cos u'):$$

which are formulæ corresponding with the original equation,

$$\sin(u' - u) = e(\sin u' - \sin u).$$

Note on the supposed Trapezoid Shape of the Sun's Corona.

By S. M. Drach, Esq.

From Mr. Proctor's "Sun as Ruler of Planetary System," coloured plate, p. 343, of Mr. Gilman's View of Eclipse of Aug. 7, 1869, I suggest that the contour *may* be the evolute of an ellipse, or the reflexion-caustic, when the reflector is an ellipse, ◇, and the radiant point in the centre (Coddington's *Optics*, p. 24, 25, fig. 22, 23, &c.). Long before Vassenius, in 1733, viz. at the great eclipse of May 1, 1706, the rose-coloured prominence was noticed by Capt. Stannyan, at Berne (first Flamstead says), and the corona, by M. Duillier, at Geneva, and also at Leipzic, *vide* Flamstead's Report, *Phil. Trans.*, No. 306, p. 2237; Hutton's *Abridgment*, v. 295; Hutton's *Mathematics of Ladies' Diaries*, i. 35–39, &c.

1875, *April* 5.